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THE REVIEW OF WAYS OF UNDERSTANDING IN PROVING CONGRUENCE OF TWO TRIANGLES

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Abstract

This study aims to reviewing ways of understanding of prospective mathematics teacher students in the process of proving the triangle congruence theorem deductively. Deductive proof is a process that is quite difficult to do if students do not know the postulates, theorems, definitions, and properties that can be used as references in the proof process. The mathematical critical thinking process needs to be reviewed to determine the relevance of students' considerations in choosing the various references needed. The study used a case study to investigate the phenomenon specifically. The participants involved in the study were five students from a university in West Java. Theory of ways of understanding is needed to examine students' understanding of postulates, theorems, definitions, and other properties that have been studied previously so that it can be known to what extent students can validate the proof process carried out. The results of the study showed that based on the ways of understanding they have, students can prove the congruence theorem of two triangles by formulating the main problems, expressing facts, choosing logical arguments, detecting information bias with different points of view, and being able to draw conclusions. Thus, in the deductive proof process, a good way of understanding is required regarding postulates, theorems, definitions, and other relevant properties to reach systematic conclusions.

Kata Kunci: Ways of Understanding; *Pembuktian Deduktif; Kongruensi Dua Segitiga*.

Penelitian ini bertujuan untuk mengkaji Ways of Understanding (WoU) *mahasiswa calon guru matematika dalam proses pembuktian teorema kongruensi segitiga secara deduktif. Pembuktian deduktif merupakan proses yang cukup sulit dilakukan apabila mahasiswa belum mengetahui postulat, teorema, definisi, dan sifat-sifat yang dapat dijadikan acuan dalam proses pembuktian. Proses tersebut perlu dikaji untuk mengetahui relevansi pertimbangan mahasiswa dalam memilih berbagai acuan yang dibutuhkan. Penelitian ini menggunakan studi kasus untuk menyelidiki pembuktian teorema secara spesifik. Partisipan yang terlibat dalam penelitian ini adalah lima orang mahasiswa dari salah satu perguruan tinggi di Jawa Barat. Teori* Ways of Understanding (WoU) *diperlukan untuk mengkaji pemahaman mahasiswa terhadap postulat, teorema, definisi, dan sifat-sifat lain yang telah dipelajari sebelumnya sehingga dapat diketahui sejauh mana mahasiswa dapat memvalidasi proses pembuktian yang dilakukan. Hasil penelitian menunjukkan bahwa berdasarkan Ways of Understanding* (WoU) *yang dimilikinya, mahasiswa mampu membuktikan teorema kongruensi dua segitiga dengan merumuskan masalah pokok, mengemukakan fakta, memilih argumen yang logis, mendeteksi kebiasan informasi dengan sudut pandang yang berbeda, dan mampu menarik kesimpulan. Dengan demikian, dalam proses pembuktian deduktif diperlukan* Ways of Understanding (WoU) *yang baik mengenai postulat, teorema, definisi, dan sifat-sifat relevan lainnya agar dapat mencapai kesimpulan yang valid dan sistematis*.

INTRODUCTION

Mathematics as a discipline that clearly relies on the thinking process is considered very good to be taught to students. It contains various aspects that are substantial for logical thinking according to patterns and rules that have been arranged in a standard way. So often the main goal of teaching mathematics is none other than to accustom students to be able to think logically, critically, and systematically (Karakoç, 2016). In geometry, theorem proof is one of the important aspects in learning mathematics, because it helps students understand basic concepts more deeply and logically. The study of geometry learning must continue to be developed so that every geometry learner is able to analyze objects into a geometric concept and can construct geometric knowledge with formal proofs (Maarif, 2016).

However, in practice, many students face difficulties in understanding and compiling theorem proofs, especially on topics such as triangle congruence, properties of plane figures, and relationships between angles (Masfingatin, Murtafiah, & Krisdiana, 2018; Lusiyana, 2024). In addition, research by Budiarto & Artiono (2019) shows that subjects at the university level make mistakes related to spatial insight, namely: interpreting images in three dimensions as images in two dimensions, for example, crossed lines are considered to intersect; not understanding the axiom that shapes can be expanded and lines can be extended; and misperception of visual processes and activities. Subjects make mistakes related to proof, namely: not being able to use axioms, definitions, theorems to solve proof problems; weak logical power; unable to use previous acquisitions; unable to change problems in questions into geometric image language; unable to understand concepts or definitions; lacking basic geometric abilities; weak spatial insight; and having no ideas to get out of the routine to solve non-routine problems.

Geometry is a mathematical system that uses deductive reasoning, based on known and accepted facts to discover new properties (Susanah, 2004). As a deductive system, the truth of a statement in geometry is proven based on logic. A theorem is a statement that must be proven true. Statements in a theorem are usually in the form of implications or biimplications. Therefore, proving a theorem means proving the truth of a mathematical sentence. Theorem statements can be divided into two parts, namely a hypothesis that shows what is known and a conclusion that shows what will be proven which involves mental action in the process (Rich, 2004). Here are some examples of mental actions, namely interpreting, conjecturing, inferring, proving, explaining, structuring, generalizing, applying, predicting, classifying, searching, and problems. However, actions are often found that can be interpreted differently depending on the context. Harel (2008) identified a tendency towards views on mathematics that developed among most teachers, namely that mathematics is viewed as subject matter (for example mathematical objects such as definitions, theorems, proofs of theorems, problems, and their solutions), not vice versa by viewing mathematics as a conceptual tool for constructing these mental objects. In essence, the two categories of knowledge (subject matter and conceptual tools) are very necessary in the context of cognitive, didactic-pedagogical, and epistemological mathematics studies.

Furthermore, Harel (2008) also stated that the two categories of knowledge should be studied in the mental acts which are one of the elements of the triadic model. Harel (2007) stated that the idea of mental acts refers to actions such as interpreting, guessing, concluding, proving, explaining, generalizing, applying, predicting, classifying, searching, and solving problems. Meanwhile, the process of proving is the process of eliminating or instilling doubt about a statement (Harel, 2007). In mathematics, the process of proving is carried out deductively, namely the logistic process to show that a statement or theorem is true based on previously accepted principles, rules, or axioms (Harel, 2007). This triadic model is the basis for the emergence of a new breakthrough regarding the definition of pedagogical mathematics where mathematics consists of two complementary subsets, namely: 1) the first subset is the set of all Ways of Understanding (WoU) is a way of understanding consisting of a collection or structure of axioms, definitions, theorems, proofs, problems, and solutions; and 2) the second subset consisting of all Ways of Thinking (WoT) is the characteristic of mental actions from the products of the first subset. Meanwhile, Ways of Thinking (WoT) is a characteristic of mental actions including how someone solves a problem, interprets a symbol or notation, proves a statement, connections between concepts, and so on. Meanwhile, Ways of Understanding (WoU) is a product of mental action that includes solutions to a symbol, problem, or concept. The difference between the two is explained by Harel (2008) through three mental actions, namely interpreting act, problem solving act, and proving act. From the perspective of Ways of Understanding, Harel (2018) stated that a concept can be understood in different ways so that it can provide benefits to change the way of understanding a concept when trying to solve a problem that is generally not within the reasoning abilities of most individuals, especially in deductive proof in Geometry. In this study, a review was conducted on thingking of process in students based on ways of understanding in proving the congruence of two triangles which was carried out deductively. The research question in this study is "how is the ways of understanding in proving the congruence of two triangles?”

METHOD

The study used a case study with the aim of revealing the phenomenon more specifically (Cresswell, 2007). The Interpretive Phenomenological Analysis (IPA) approach aims to interpret and interpret a phenomenon based on human experience (Eatough & Smith, 2017). Where IPA is closely related to phenomenology and hermeneutics which focus on a person's experience. As Ricoeur (1986) said, it is necessary to combine the study of experience and the study of meaning and meaning with that experience because they complement each other. This was chosen to reveal the variety of meanings and describe the mathematical thinking process of students in solving non-routine mathematical problems on number pattern material. Participants in this study involved five prospective mathematics teacher students from a university in West Java, Indonesia.

The data analysis techniques used in this study are based on the stages developed by Creswell (2007), namely data managing, reading-memoing, describing-classifying-interpreting, and representing-visualizing. Data managing, namely organizing data into computer files for analysis, transcribing student recordings and interviews, and typing observation notes. Reading memoring, namely reading and interpreting the collected data and providing notes or memos on the edge of field notes or transcripts or under photos to assist in the initial process of data exploration. Describing-classifying-interpreting, namely forming codes or categories representing the core of data analysis. Researchers build detailed descriptions, develop themes or dimensions, and provide interpretations based on their own views or perspectives in the literature. Representing-visualizing, namely representing the results of data analysis.

The findings of the study interpret by used the theory of ways of understanding to examine the postulates, definitions, theorems, and properties that are relevant to use in the process of proving the concurrency of two triangles. The questions as follows: 1) In this known that $\overline{HF} \perp \overline{BD}$, $\overline{HG} \perp \overline{AC}$, and $\overline{HF} \cong \overline{HG}$, prove that $\overline{AG} \cong \overline{DF}$; 2) In this known $\measuredangle A \cong \measuredangle C, \overline{AB} \cong \overline{CB}$, prove that $\triangle CBE \cong \triangle ABD$; 3) In this known that ABCDEF consecutive hexagons, prove that $\triangle ABD \cong \triangle AFD$.

FINDINGS AND DISCUSSIONS

Based on the questions given to the participants, answers were obtained to review the mathematical thinking process carried out based on the participants' ways of understanding in determining relevant postulates, theorems, and properties. The following shows the answers to each question:

1. In this known that $\overline{HF} \perp \overline{BD}$, $\overline{HG} \perp \overline{AC}$, and $\overline{HF} \cong \overline{HG}$, prove that $\overline{AG} \cong \overline{DF}$

Participants are asked to prove that line segment AG is congruent to line segment DF based on known information, namely that line segment HF is perpendicular to line segment BD, line segment HG is perpendicular to line segment AC, and line segment HF is congruent to line segment HG. The following shows a geometric illustration of the given question in the Figure 1.

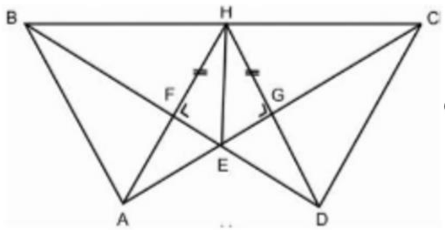

Figure 1. Geometric illustration

Based on the geometric illustration in Figure 1, it is shown that there are several triangles formed, including $\triangle ABE$, $\triangle EDC$, $\triangle ABC$, $\triangle BDC$, $\triangle BEH$, $\triangle CEH$, $\triangle EDC$, $\triangle BEC$, $\triangle BFH$, $\triangle CGH$, $\triangle EFH$, $\triangle EGH$, $\triangle DGC$, $\triangle ABF$, $\triangle AEF$, $\triangle EDG$. In the given question, it is requested to prove that $\overline{AG} \cong \overline{DF}$, therefore from the triangles formed, it is necessary to choose a triangle that can lead to the proof process. The following is the answer from one of the participants shown in Figure 2.

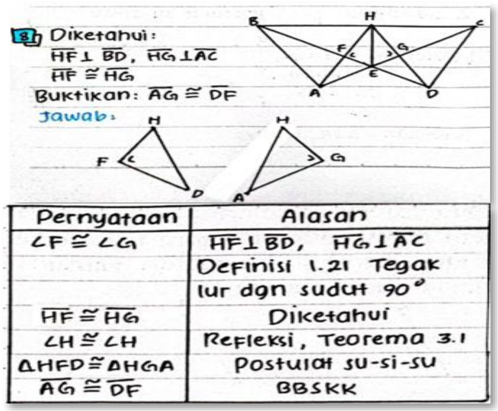

Figure 2. Participants' Answers

Based on Figure 2, participants identified the information known in the question, namely $\overline{HF} \perp \overline{BD}$, $\overline{HG} \perp \overline{AC}$, and $\overline{HF} \cong \overline{HG}$ then draw geometric shapes like those in the building $ABCD$ in which there are many triangles. Participants then determine the triangles that support the proof process of $\overline{AG} \cong \overline{DF}$, namely $\triangle DFH$ and $\triangle AGH$. There are many triangles found in the form of $ABCD$, $\triangle DFH$ and $\triangle AGH$ chosen by participants because it collects information contained in the questions. For this reason, participant have a good critical thinking process to consider the steps needed in the proof process. In Figure 2 it is known that $\overline{AG} \cong \overline{DF}$ be in the part of $\triangle DFH$ and $\triangle AGH$. Therefore, participants can prove $\overline{AG} \cong \overline{DF}$ through ways of understanding the nature of the corresponding parts of congruent triangles are congruent by first showing that $\triangle DFH \cong \triangle AGH$ by using the angle-side-angle postulate. Participants indicated that $\measuredangle F \cong \measuredangle G$ by using the definition of two perpendicular lines if the two lines intersect to form a right angle so that $m \measuredangle F = 90^0$ and $m \measuredangle G = 90^0$. Therefore, it is proven $\measuredangle F$ congruent with $\measuredangle G$ because they have the same angle size. Next the participants indicated that the line segment $\overline{HF} \cong \overline{HG}$ based on previously known information. Participants also show that $\measuredangle H$ congruent with itself based on reflective properties. Based on the congruence on $\measuredangle F \cong \measuredangle G, \overline{HF} \cong \overline{HG}$, dan $\measuredangle H \cong \measuredangle H$, then the participants conclude that angle side angle postulate in $\triangle$

DFH and $\triangle AGH$ is fulfilled. Thus $\triangle DFH \cong \triangle AGH$ so that $\overline{AG} \cong \overline{DF}$ for reasons considered by the participants that corresponding parts of congruent triangles are congruent.

2. In this known $\angle A \cong \angle C$, $\overline{AB} \cong \overline{CB}$, prove that $\triangle CBE \cong \triangle ABD$

Participants are asked to prove the congruence of two triangles, namely △CBE and △ABD. From the questions given, participants write down the known information to compile the steps that can be taken to prove △CBE and △ABD. Based on the known information, angle A is congruent to angle C and side AB is congruent to side CB. Based on the known information, participants need to find other supporting conditions to arrive at the postulate to obtain the conclusion that △CBE is congruent to △ABD. The participants' answers are shown in Figure 3.

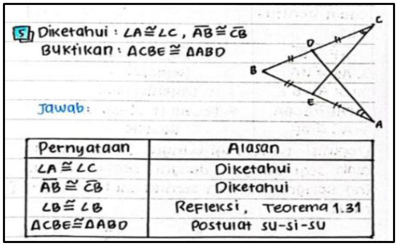

Figure 3. Participants' Answers

Based on the answers shown, participants ranked the known information, $\angle A \cong \angle C$ and $\overline{AB} \cong \overline{CB}$. Next, participants demonstrated ways of understanding about the reflexive properties that apply to angle congruence to lead to the angle-side-angle postulate to prove that △CBE and △ABD. Participants chose the postulate because it supported the evidence based on previously available information. Participants indicated that $\angle B$ congruent to itself based on the reflective property. Thus, the angle-side-angle postulate is fulfilled and △CBE ≅ △ABD.

3. In this known that ABCDEF consecutive hexagons, prove that $\triangle ABD \cong \triangle AFD$.

Participants were asked to prove that $\triangle ABD$ congruent with $\triangle AFD$ from $ABCDEF$ consecutive hexagon. In this question, no information is provided relating to the proof process, so participants need to think critically and demonstrate ways of understanding related to the postulates, definitions and properties needed to lead to proof. The following answer from participants is shown in Figure 4.

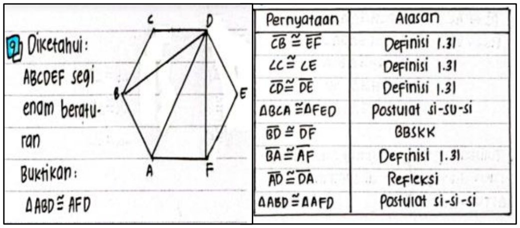

Figure 4. Participants' Answer

The proof process carried out by participants as in Figure 4 based on the ways of understanding they have, begins by showing that $\overline{CB}$ congruent with $\overline{EF}$, $\angle C$

congruent with $\measuredangle E$, $\overline{CD}$ congruent with $\overline{DE}$ using the definition that the regular polygon is a polygon with all sides and all angles congruent. Thus, $\triangle BCD$ congruent with $\triangle DEF$ based on the postulate of the side-angle-side. The researchers then asked why the participants used congruence between $\triangle BCD$ with $\triangle DEF$? The participant then answered that the use of congruence in the triangle was so that the proof process could be directed towards the elements contained in $\triangle ABD$ and $\triangle AFD$. From these answers, participants were able to explain the reasons for the proof steps taken so that it can be said that the participants' mathematical critical thinking process was good. Next to prove that $\triangle ABD \cong \triangle AFD$, participants use the side-by-side postulate and showed that: 1) $\overline{BD} \cong \overline{DF}$ with the reason by corresponding parts of congruent triangles are congruent based on the previous proof $\triangle BCD \cong \triangle DEF$; 2) $\overline{BA} \cong \overline{AF}$ based on using the definition that the regular polygon is a polygon with all sides and all angles congruent; $\overline{AD} \cong \overline{DA}$ based on reflective properties. From the fulfillment of the side-side postulate, it is thus proven that $\triangle ABD \cong \triangle AFD$.

The study revealed several key findings regarding the relationship between “ways of understanding” and students' success in deductive proofs: 1) Conceptual Understanding as the Basis for Proof: A conceptual understanding of basic geometric concepts, such as points, lines, angles, and triangles, plays a significant role in deductive proofs. Students who have a deep understanding of these concepts are better able to identify the important elements in a problem and relate them to relevant theorems or definitions. For example, in proving that the sum of the angles in a triangle is 180 degrees, students need to understand the definition of a triangle, the concept of angles, and the relationship between those angles and parallel lines. Students who understand these concepts deeply are better able to construct logical and valid arguments; 2) Procedural Understanding to Apply Theorems: Students also need procedural understanding, which is the ability to use the rules of geometry in proofs. This includes skills such as drawing accurate diagrams, identifying given information, and applying logical steps to reach conclusions. The findings suggest that students who understand the procedures of a proof do not simply follow the steps mechanically, but are also able to explain the reasoning behind each step. For example, in proving the similarity of two triangles using the angle-side-angle (ASA) criterion, students need to understand why the criterion is sufficient to conclude the similarity; 3) Thinking Strategies and Deductive Reasoning: In addition to conceptual and procedural understanding, students also need good thinking strategies to build deductive arguments. These abilities include deductive reasoning, which is the ability to draw conclusions based on given premises, and the ability to recognize patterns in geometric problems. This study found that students who have good thinking strategies tend to be more flexible in dealing with proof problems. They can explore various approaches to proving a theorem, such as using the method of contradiction or mathematical induction; 4) The Role of Visualization in Understanding Problems: Visualization is an important aspect in understanding and solving geometric problems. Students who can draw diagrams well tend to be more easily able to understand the relationships between elements in geometry. In addition, accurate diagrams can help students explore geometric relationships that may not be immediately apparent. For example, in proving that the diagonals of a rectangle bisect each other, students need to draw clear diagrams and understand the properties of rectangles, such as symmetry and equality of side lengths.

The deductive proof is an essential aspect of learning flat geometry. This process involves logical thinking and the use of established rules to prove a statement or theorem. In the context of mathematics education, deductive proof not only facilitates conceptual understanding but also develops students' critical thinking skills. However, the effectiveness of deductive proof is highly dependent on how students understand the material on which the proof is based. One important approach to consider is "ways of understanding.

The ways of understanding refer to the various ways in which students interpret concepts and principles in geometry. This understanding includes not only declarative knowledge, such as definitions and theorems, but also procedural and strategic knowledge, such as how to use these rules in proofs. In this section, we will discuss research findings that demonstrate the importance of "ways of understanding" in the deductive proof process and its implications for learning flat geometry. The ways of understanding are an important component in the deductive proof process, especially in geometry material. According to Karakoç (2016), ways of understanding is very important in helping students develop analytical skills needed to solve complex problems and understand mathematical concepts in depth. This ability not only improves students' academic performance, but also prepares them to face real-world challenges that require critical and logical thinking. In line with Ennis (2011), the development of ways of understanding in mathematics learning allows students to evaluate information logically, make strong arguments, and make decisions based on in-depth analysis of available data. These skills are very important in building mathematical competence that focuses not only on procedures, but also on conceptual understanding. This finding is in line with the research results of Budiarto (2019) which revealed that Geometry problems based on basic geometry skills are consecutively problems related to logic skills, drawing skills, visual skills, verbal skills, and applied skills. If we review geometry learning using analytical presentations, the problem of deductive use is at the top such as the problem of proof, then the problem of perception, misconceptions about visual processes and activities, and finally the problem of using procedures, concepts, and principles. Furthermore, based on the findings presented previously, ways of understanding in geometry allows students to not only understand the properties of geometric shapes, but also be able to prove theorems and develop strong logical arguments. The use of ways of understanding in geometry learning encourages students to explore various approaches in solving problems and strengthen their analytical skills (Prayitno, 2018; Santosa, 2013; Yerizon, 2011). This statement is in line with Ikhsan, Munzi, & Fitria (2017) who stated ways of understanding in proving geometric theorems allows students to not only passively accept theorems, but also to understand the underlying logical steps. The proof process requires an in-depth evaluation of each argument, which teaches students to develop analytical skills that are important in mathematics and other disciplines. Proving geometric theorems requires in-depth critical thinking skills, because students must be able to identify assumptions, construct logical arguments, and evaluate conclusions systematically. Developing these skills is essential to help students understand mathematical structures more thoroughly and deeply (Rasiman, 2015; Suandi, 2017). Ways of understanding play an important role in improving students' critical mathematical thinking skills, especially in proving geometric theorems. A deep understanding of basic concepts allows students to connect various mathematical ideas logically and construct valid and systematic evidence (Aiyub, 2023).

According to Harel & Sowder (1998) said that ways of understanding refer to how individuals understand certain mathematical concepts or objects. It includes the meaning and knowledge that a person acquires through learning experiences, and how they interpret these concepts. In the context of mathematics learning, ways of understanding are the result of a learning process that involves thinking activities and interacting with various mathematical problems or concepts. Harel (2008) also emphasized that ways of understanding are closely related to how students develop critical thinking skills in mathematics. Deep understanding allows students to connect various mathematical ideas, which is very important in the process of proof and problem solving, including in geometry.

Thus, the potential for epistemological barriers to learning due to the inability to resolve contexts that are different from those previously encountered can be minimized (Prihandhika, et al., 2020). Ways of understanding play an important role in building critical mathematical thinking skills, especially in proving geometric theorems. With a deep understanding, students can better relate geometric concepts, evaluate arguments logically, and produce stronger and more systematic evidence" (Harel, 2008). Ways of understanding help students build a strong foundation in understanding mathematical concepts, especially in geometry. This deep understanding improves their competencies, which are essential in the process of proving theorems, as they can better identify important elements, evaluate logical arguments, and construct valid proofs.

CONCLUSIONS

The deductive proof is an essential aspect of learning flat geometry. This process involves logical thinking and the use of established rules to prove a statement or theorem. In the context of mathematics education, deductive proof not only facilitates conceptual understanding but also develops students' critical thinking skills. However, the effectiveness of deductive proof is highly dependent on how students understand the material on which the proof is based. One important approach to consider is "ways of understanding. The Ways of understanding refers to the various ways in which students interpret concepts and principles in geometry. This understanding includes not only declarative knowledge, such as definitions and theorems, but also procedural and strategic knowledge, such as how to use these rules in proofs. In this section, we will discuss research findings that demonstrate the importance of ways of understanding in the deductive proof process and its implications for learning flat geometry.

The ways of understanding play an important role in the process of proving the congruence theorem of two triangles. Ways of understanding allows students to analyze various elements of a triangle logically, identify similarities between sides and angles, and evaluate arguments used in the proof. On the other hand, ways of understanding provide a foundation for a deep understanding of geometric concepts, which facilitates students in arranging proofs systematically and coherently. By developing these two aspects, students can produce more accurate, logical, and structured theorem proofs, which ultimately strengthen their overall mathematical abilities.

The ways of understanding play a very important role in deductive proofs in flat geometry. Conceptual understanding, procedural skills, thinking strategies, and visualization abilities all contribute to students' success in proofs. Therefore, teachers need to design learning that allows students to develop these different ways of understanding. With the right approach, students will not only be able to perform deductive proofs well, but will also develop critical thinking skills that will be useful in various aspects of their lives. Here are some important implications for learning flat geometry: 1) Strengthening Conceptual Understanding: Teachers need to ensure that students have a strong understanding of basic geometric concepts before introducing deductive proofs. This can be done through discussions, explorations, and activities that allow students to construct their own understanding. For example, students can be asked to explore the relationship between angles in various geometric shapes using geometric software; 2) Developing Procedural Skills: In addition to conceptual understanding, students also need to be trained to apply geometric procedures in proofs. Teachers can provide exercises that involve different types of proofs, such as direct proof, contradiction, and induction. These exercises should include varying levels of difficulty to ensure that students have a deep understanding of the procedures used; 3) Focus on Deductive Thinking Strategies: Teachers need to help students develop effective thinking strategies in deductive proofs. This can be done by teaching different approaches to proving a theorem, such as looking for patterns, using analogies, or starting from the conclusion you want to reach. In addition, teachers can provide examples of proofs involving complex deductive reasoning to train students' critical thinking skills; 4) Encourage the Use of Visualization: Visualization can be a very useful tool in geometry learning. Teachers can encourage students to draw accurate diagrams and use visual aids, such as dynamic geometry software, to help them understand geometric relationships. In addition, teachers can provide assignments that involve interpreting diagrams to train students' visualization skills; 5) Evaluation and Feedback: Good evaluation can help teachers understand students' level of understanding and provide constructive feedback. Teachers can use a variety of evaluation methods, such as written tests, group discussions, and proof presentations, to measure students' abilities in deductive proof. The feedback given should focus on aspects that need improvement, such as errors in logic or poor conceptual understanding.

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